A GRAPH WHOSE VERTICES ARE ALL THE DIVISORS OF A POSITIVE INTEGER: FUNDAMENTALS

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Abstract.
Given n a positive integer, the graph Div(n) is defined as follows. The vertices of Div(n) are all of the divisors of n with two vertices u and v being adjacent if either u divides v and $v = up$ with p a prime or v divides u and $u = vp$ with p a prime. A characterization of the structure of Div(n) is derived and Hasse diagrams are employed to illustrate this structure. The relation of Div(n) to posets, lattices, algebras (Boolean, Post, and P), and digital networks is discussed. Comparisons and properties of their associated graphs are investigated.

1. Introduction

Let n be a positive integer greater than one written as a product of primes $p_1 < p_2 < \cdots < p_k$, that is,

$$n = p_1^{n_1} p_2^{n_2} \ldots p_k^{n_k}$$

so that n is the product of $n_1 + n_2 + \cdots + n_k = L$ primes to the first power.

Next partition the divisors of n into *levels* where the divisors on level $e \geq 1$ are the divisors of n having exactly e prime, not necessarily distinct, factors. Define *level* 0 to be the lowest level containing only the integer 1. For example, the partitions of 8 = 2^3, 12 = $2^2 3$, and 30 = $2 \cdot 3 \cdot 5$ into levels are, respectively, shown in Table 1.1.

level	n = 8	n = 12	n = 30
3	8	12	30
2	4	4 6	6 10 15
1	2	2 3	2 3 5
0	1	1	1

Table 1.1 The levels of Div(n)

The following graph is defined based on any given positive integer $n \geq 1$.